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An Exact Analysis of Group Velocity for Propagation Modes in Optical Fibers

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***Abstract*—A method for calculating exactly the group velocity of propagation modes in optical fibers is proposed. In this analysis, optical fibers can contain uniaxial and dispersive media. The group velocity obtained by using the scalar approximate analysis is compared numerically with the rigorous group velocity computed by this method for square-law index optical fibers.**

I. Introduction

The SCALAR approximation method is one of the most widely used techniques in optical fiber analysis because of short computing time and simple treatment. The method, however, has much error in the near cutoff region [1], [2], and the inaccuracies of group velocity are of practical importance for the analysis of pulse broadening, particularly in optical fibers which have few propagation modes. A method is needed for calculating exactly the group velocity of propagation modes in various optical fibers. There are several approximate methods giving the group velocity, i.e., a computational method based on the WKB theory [3], a method using the scalar finite-element analysis [4], a practical method using the scalar multilayer analysis [5], and a method with the vector multilayer analysis and the integral expression for the group velocity in the scalar analysis [6]. Kharadly [7] calculated the exact group velocity of the dielectric-tube waveguides constituted by three layers without material dispersion. To the authors' knowledge, however, a practical method giving the exact group velocity has not yet been proposed.

It is the purpose of this paper to describe a method for computing rigorously the group velocity of propagation modes in optical fibers without numerical differentiation, including uniaxial and dispersive material. The group velocity is determined by using the vector multilayer approximation and the integral expression for the group velocity in vector analysis. The calculated results have only the error caused by the multilayer approximation of index distribution, and are exact for the staircase index optical

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fibers. Computing the group velocity requires relatively short running time on the computer, because the integration in the expression for the group velocity can be analytically obtained with little additional calculation. In the present paper, the scalar approximate solutions are compared numerically with the vector (rigorous) solutions for various modes of propagation in cylindrical fibers of square-law index distributions.

II. Vector Multilayer Analysis

Since exact analysis for graded-index optical fibers is difficult and time-consuming, appropriate approximation techniques have been developed for typical multimode optical fibers. In case the optical fibers have few propagation modes, the error due to the approximation is of practical importance for calculating pulse broadening. On the other hand, the computing time for vector analysis can be made relatively short because of small numbers of propagation modes. In this paper, therefore, a method for calculating exactly the group velocity of propagation modes is described.

It is assumed that the permittivity $\hat{\epsilon}$ of the fiber depends only upon the distance r from the axis, and the permeability is equal to that of vacuum μ_0. The permittivity tensor of the media of optical fibers is assumed to be the form

$$\hat{\epsilon}=\begin{bmatrix}\epsilon_t & 0 & 0\\ 0 & \epsilon_t & 0\\ 0 & 0 & \epsilon_z\end{bmatrix}=\begin{bmatrix}\epsilon_0 n_t^2 & 0 & 0\\ 0 & \epsilon_0 n_t^2 & 0\\ 0 & 0 & \epsilon_0 n_z^2\end{bmatrix}. \quad (1)$$

The refractive-index profile in the core region is represented approximately by a stratified multilayer structure. Eliminating the transverse electromagnetic field from Maxwell's equations, we get the following wave equations in the ith layer:

$$\frac{d^2e_z}{dr^2}+\frac{1}{r}\frac{de_z}{dr}+\left\{s_i^2(k^2n_{ti}^2-\beta^2)-\frac{m^2}{r^2}\right\}e_z=0 \quad (2)$$

$$\frac{d^2h_z}{dr^2}+\frac{1}{r}\frac{dh_z}{dr}+\left\{(k^2n_{ti}^2-\beta^2)-\frac{m^2}{r^2}\right\}h_z=0 \quad (3)$$

where $k=\omega\sqrt{\epsilon_0\mu_0}$, $s_i=n_{zi}/n_{ti}$, and β is a propagation constant of the guided mode; the longitudinal components of the electromagnetic fields, i.e., E_z and H_z, are stated as

$$E_z(r,\theta)=e_z(r)\begin{Bmatrix}\cos m\theta\\ \sin m\theta\end{Bmatrix}$$

$$H_z(r,\theta)=h_z(r)\begin{Bmatrix}\cos m\theta\\ \sin m\theta\end{Bmatrix}. \quad (4)$$

The solutions of (2) and (3) are represented rigorously in terms of Bessel functions, and the longitudinal and circumferential components of the electromagnetic fields are expressed as

$$\begin{bmatrix}e_z\\ \dfrac{e_\theta}{j}\\ \dfrac{\omega\mu_0}{\beta}h_z\\ \dfrac{\omega\mu_0}{j\beta}h_\theta\end{bmatrix}=\boldsymbol{P}_i(r)\begin{bmatrix}A_i\\ B_i\\ C_i\\ D_i\end{bmatrix} \quad (5)$$

where

$$\boldsymbol{P}_i(r)=\begin{bmatrix}Z_m(s_iu_ir) & \bar{Z}_m(s_iu_ir) & 0 & 0\\ \dfrac{m\beta}{\eta^2 r}Z_m(s_iu_ir) & \dfrac{m\beta}{\eta^2 r}\bar{Z}_m(s_iu_ir) & \dfrac{\beta u_i}{\eta^2}Z_m'(u_ir) & \dfrac{\beta u_i}{\eta^2}\bar{Z}_m'(u_ir)\\ 0 & 0 & Z_m(u_ir) & \bar{Z}_m(u_ir)\\ -\dfrac{s_iu_ik^2n_{ti}^2}{\eta^2\beta}Z_m'(s_iu_ir) & -\dfrac{s_iu_ik^2n_{ti}^2}{\eta^2\beta}\bar{Z}_m'(s_iu_ir) & -\dfrac{m\beta}{\eta^2 r}Z_m(u_ir) & -\dfrac{m\beta}{\eta^2 r}\bar{Z}_m(u_ir)\end{bmatrix} \quad (6)$$

and A_i, B_i, C_i, and D_i are unknown coefficients, $\eta^2=k^2n_{ti}^2-\beta^2$, and Z_m and $\bar{Z}_m$ are signified as follows:

i) $Z_m=J_m$, $\bar{Z}_m=N_m$ for $u_i^2=k^2n_{ti}^2-\beta^2>0$.

ii) $Z_m=I_m$, $\bar{Z}_m=K_m$ for $-u_i^2=k^2n_{ti}^2-\beta^2<0$.

By using (5) and the continuity of the tangential fields e_z, h_z, e_θ, and h_θ at each boundary $r=a_i$ of the layer, the unknown coefficients in the ith layer can be expressed in terms of the coefficients in the first layer as follows:

$$\begin{bmatrix}A_i\\ B_i\\ C_i\\ D_i\end{bmatrix}=\boldsymbol{P}_i^{-1}(a_{i-1})\boldsymbol{P}_{i-1}(a_{i-1})\cdots\boldsymbol{P}_2^{-1}(a_1)\boldsymbol{P}_1(a_1)\begin{bmatrix}A_1\\ B_1\\ C_1\\ D_1\end{bmatrix}. \quad (7)$$

Since the electromagnetic fields are finite at $r=0$ and ∞, the coefficients B_1, D_1, A_{N+1}, C_{N+1} must satisfy

$$B_1=D_1=A_{N+1}=C_{N+1}=0 \quad (8)$$

where the $N+1$th layer is the cladding. The relation between coefficients in the first layer and the cladding is derived from (7) and (8) as follows:

$$\begin{bmatrix}0\\ B_{N+1}\\ 0\\ D_{N+1}\end{bmatrix}=\boldsymbol{P}\begin{bmatrix}A_1\\ 0\\ C_1\\ 0\end{bmatrix} \quad (9)$$

where

$$\boldsymbol{P}=\begin{bmatrix}P_{11} & P_{12} & P_{13} & P_{14}\\ P_{21} & P_{22} & P_{23} & P_{24}\\ P_{31} & P_{32} & P_{33} & P_{34}\\ P_{41} & P_{42} & P_{43} & P_{44}\end{bmatrix}=\boldsymbol{P}_{N+1}^{-1}(a_N)\boldsymbol{P}_N(a_N)\cdots\boldsymbol{P}_2^{-1}(a_1)\boldsymbol{P}_1(a_1). \quad (10)$$

In order that nontrivial solutions of (9) exist

$$P_{11}P_{33} - P_{13}P_{31} = 0 \tag{11}$$

must be satisfied. The propagation constants can be determined by the eigenvalue equation (11), and the coefficients in each layer can be obtained from (7).

We shall derive next an integral expression for the group velocity of the guided modes. The variational expression for the propagation constant is given as [8], [9]

$$\beta = \frac{\int_S \{\omega E^* \cdot \hat{\epsilon} \cdot E + \omega H^* \cdot \hat{\mu} \cdot H - jH^* \cdot \nabla_t \times E + jE^* \cdot \nabla_t \times H\} ds - j\int_C \{H^* \times E\} \cdot n \, dl}{\int_S \{H^* \cdot i_z \times E - E^* \cdot i_z \times H\} ds} \tag{12}$$

Application of an analogy with the Hellmann–Feynman theorem [10] yields an exact expression for the group velocity

$$\frac{d\omega}{d\beta} = \frac{\int_S \{i_z \cdot (E \times H^* + E^* \times H)\} ds}{\int_S \left\{ E^* \cdot \frac{d(\omega\hat{\epsilon})}{d\omega} \cdot E + H^* \cdot \frac{d(\omega\hat{\mu})}{d\omega} \cdot H \right\} ds} \tag{13}$$

whose denominator and numerator indicate the total energy stored within the unit length of the waveguide and the time-average power flow along the waveguide, respectively. This expression can be used not only for isotropic material but for anisotropic and dispersive material. The refractive-index of the stratification approximation for analyzing graded-index waveguides is constant in each layer. Therefore, the expression for the group velocity, (13), can be rewritten in the form

$$\frac{d\omega}{d\beta} = V_g = C \frac{\sum_i M_i}{\sum_i L_i} \tag{14}$$

where C is the velocity of light in vacuum, and

$$M_i = \frac{2\beta^2}{\eta^2}\left[\frac{s_i^2 k^2 n_{ti}^2}{\beta^2} I_{1i} + I_{2i}\right] + 2\left(\frac{\beta}{\eta^2}\right)^2 \left[\frac{k^2 n_{ti}^2}{\beta^2} I_{3i} + I_{4i} + m\left(1 + \frac{k^2 n_{ti}^2}{\beta^2}\right) I_{5i}\right]$$

$$L_i = \left[\frac{k}{\beta}\left(N_{zi} + \frac{s_i^2\beta^2}{\eta^2} N_{ti}\right) + \frac{\beta}{k}\frac{s_i^2\beta^2}{\eta^2}\left(\frac{k^2 n_{ti}^2}{\beta^2}\right)^2\right] I_{1i} + \left[\frac{k}{\beta}\frac{\beta^2}{\eta^2} N_{ti} + \frac{\beta}{k}\left(1 + \frac{\beta^2}{\eta^2}\right)\right] I_{2i}$$
$$+ \left[\frac{k}{\beta}\left(\frac{\beta}{\eta^2}\right)^2 N_{ti} + \frac{\beta}{k}\left(\frac{k^2 n_{ti}^2}{\eta^2}\right)^2 \frac{1}{\beta^2}\right] I_{3i} + \left[\frac{k}{\beta}\left(\frac{\beta}{\eta^2}\right)^2 N_{ti} + \frac{\beta}{k}\left(\frac{\beta}{\eta^2}\right)^2\right] I_{4i}$$
$$+ 2m\left(\frac{\beta}{\eta^2}\right)^2 \left[\frac{k}{\beta} N_{ti} + \frac{\beta}{k}\left(\frac{k^2 n_{ti}^2}{\beta^2}\right)\right] I_{5i}$$

$$N_{ti} = n_{ti}^2 - 2n_{ti}\lambda \frac{dn_{ti}}{d\lambda}$$

$$N_{zi} = n_{zi}^2 - 2n_{zi}\lambda \frac{dn_{zi}}{d\lambda}$$

$$I_{1i} = \int_{a_{i-1}}^{a_i} \{A_i Z_m(s_i u_i r) + B_i \bar{Z}_m(s_i u_i r)\}^2 r \, dr$$

$$I_{2i} = \int_{a_{i-1}}^{a_i} \{C_i Z_m(u_i r) + D_i \bar{Z}_m(u_i r)\}^2 r \, dr$$

$$I_{3i} = \left[s_i u_i r \{A_i Z_m(s_i u_i r) + B_i \bar{Z}_m(s_i u_i r)\} \cdot \{A_i Z_m'(s_i u_i r) + B_i \bar{Z}_m'(s_i u_i r)\}\right]_{r=a_{i-1}}^{r=a_i}$$

$$I_{4i} = \left[u_i r \{C_i Z_m(u_i r) + D_i \bar{Z}_m(u_i r)\} \cdot \{C_i Z_m'(u_i r) + D_i \bar{Z}_m'(u_i r)\}\right]_{r=a_{i-1}}^{r=a_i}$$

$$I_{5i} = \left[\{A_i Z_m(s_i u_i r) + B_i \bar{Z}_m(s_i u_i r)\} \cdot \{C_i Z_m(u_i r) + D_i \bar{Z}_m(u_i r)\}\right]_{r=a_{i-1}}^{r=a_i}.$$

Since the integration I_{ji} can be analytically obtained and the coefficients (A_i, B_i, C_i, D_i) are given by solving the eigenvalue equation, the group velocity is determined immediately by using (14) with little additional calculation. In this vector analysis, calculating the group velocity and the propagation constant requires relatively short running time on the computer, which is about twice as large as that in the scalar multilayer analysis [5].

III. Numerical Results

The scalar approximation solutions of the group velocity are compared numerically with the results of the vector multilayer analysis to estimate the accuracy of the scalar approximation techniques. The form of the refractive-index function to be considered is given by the following cladded-parabolic distribution:

$$n(r)^2 = \begin{cases} n_1^2\left[1 - 2\Delta\left(\frac{r}{a}\right)^2\right], & r \leq a \\ n_1^2(1 - 2\Delta) \equiv n_2^2, & r > a \end{cases} \tag{15}$$

where a is the radius of the core, n_1 and n_2 the refractive-indices at the center axis and the cladding of the fiber, respectively, and $\Delta \simeq (n_1 - n_2)/n_1$ represents the relative refractive-index difference between the core n_1 (at the center axis) and the cladding n_2. The normalized group velocity $(V_g/C - 1/n_1)/(1/n_2 - 1/n_1)$ is computed numerically by the vector rigorous analysis and scalar approximation analysis [5] for various modes of propagation and for various normalized frequency T which is defined by

$$T^2 = 2k^2 \int_{n(r) > n_2} \{n(r)^2 - n_2^2\} r \, dr. \tag{16}$$

In both analyses, the permittivity distribution in the core region is represented approximately by multiple layers of different constant. In the following numerical evaluation, the core region is divided into fifty layers. One of the interesting features obtained from the numerical calculations is that the error of the normalized group velocity is approximately proportional to the refractive-index difference Δ. Therefore, the estimation of the group velocity error is given by the following quantity:

$$e_s = \frac{|V_{gs}/C - V_{gex}/C|}{(1/n_2 - 1/n_1)\Delta} \tag{17}$$

where V_{gex} and V_{gs} indicate the group velocity calculated by the vector rigorous analysis and the scalar approximation analysis, respectively. The error due to the multilayered approximation for both analyses is of the order of 10^{-3} or less in the case of fifty layers.

Fig. 1 shows the error of the normalized group velocity versus the normalized frequency T for various HE, EH, and TM modes. The magnitude of the error decreases as the frequency increases except near the cutoff region, and the maximum value of the error e_s is about unity. These results are similar to those for the step-index fiber [2]. Since the characteristic equations for TE modes are the same for both vector and scalar analyses, TE modes have no error of the group velocity.

The group velocity differences between the corresponding vector modes for each linearly polarized mode (LP mode) in the scalar analysis are of importance for designing fibers with few propagation modes, e.g., dual-mode optical fibers [11]. Fig. 2(a) and (b) shows, respectively, the group velocity differences between the vector modes equivalent to LP_{11} and LP_{21} modes. The value e_v in Fig. 2 indicates the normalized group velocity difference divided by the refractive-index difference Δ. The group velocity difference decreases with the frequency T except near the cutoff region and its maximum value is about unity.

The group velocity error e_s for the scalar analysis and the group velocity difference e_v between vector modes forming a LP mode can be rewritten in the group delay

$$e_\tau/L \simeq \frac{n_1}{C}\Delta^2 e_{\substack{s\\v}} \simeq 5.0\times10^3\Delta^2 e_{\substack{s\\v}} \ (\text{ns/km}) \tag{18}$$

where L indicates the propagation distance. At the maximum value $e_{\substack{s\\v}} = 1$, the group delay e_τ is evaluated as follows:

$$e_\tau/L \simeq 0.5(\text{ns/km}) \qquad \text{for } \Delta = 0.01$$
$$5(\text{ps/km}) \qquad \text{for } \Delta = 0.001.$$

Therefore, the maximum group-delay error in the scalar analysis and the maximum group-delay difference between the corresponding vector modes for each LP mode are about 0.5 (ns/km) for typical multimode optical fibers and 5 (ps/km) for usual single-mode optical fibers. The delay error and difference become maximum near the cutoff region. For the practical multimode fibers, modes near cutoff have little effect on the fiber bandwidth, because they usually have much loss and little excited power [5]. It can be concluded that this computation method is useful for calculating the group delay of modes near cutoff and the bandwidth of fibers with few propagation modes, e.g., single-mode fibers and dual-mode fibers [11].

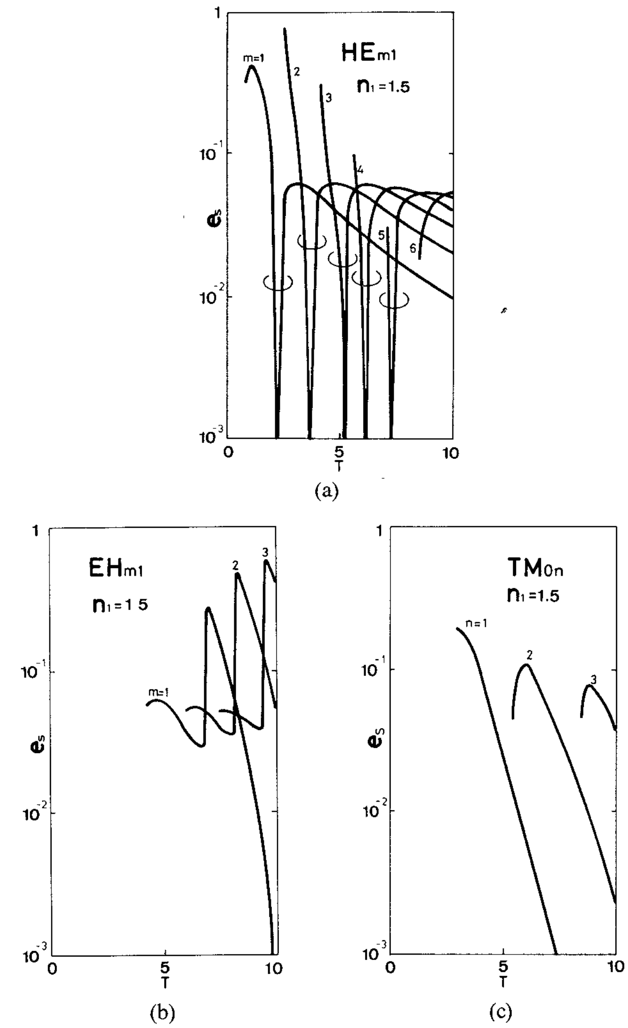

Fig. 1. The error in the group velocity due to the scalar approximation for (a) HE_{m1} mode, (b) EH_{m1} mode, and (c) TM_{0n} mode. $V = \sqrt{2}\,T = k\,an_1\sqrt{2\Delta}$.

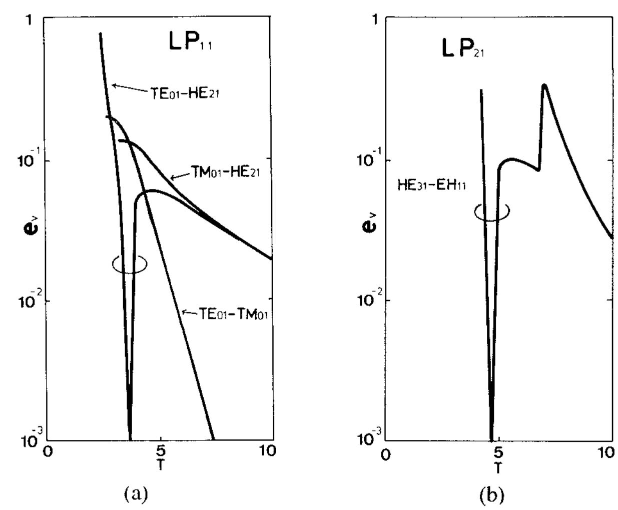

Fig. 2. The group-velocity differences between the corresponding vector modes for the linearly polarized modes, (a) LP_{11} and (b) LP_{21}. $V = \sqrt{2}\,T = k\,an_1\sqrt{2\Delta}$.

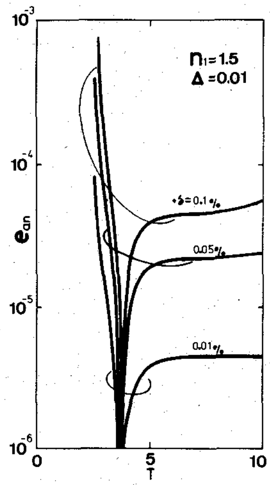

Fig. 3. The group-velocity deviations caused by uniaxial material. $V = \sqrt{2}\,T = k\,a n_1\sqrt{2\Delta}$.

The square-law index fibers with uniaxial material, which is caused by the pressure on fibers, are analyzed to clarify deviations from the group velocity expected for the isotropic square-law index fibers. The refractive-index distribution is assumed to be

$$n(r) = \begin{bmatrix} n_t(r) & 0 & 0 \\ 0 & n_t(r) & 0 \\ 0 & 0 & n_z(r) \end{bmatrix} \tag{19}$$

where

$$n_t(r)^2 = \begin{cases} n_1^2\left[1 - 2\Delta\left(\frac{r}{a}\right)^2\right], & r \leqslant a \\ n_1^2(1-2\Delta) \equiv n_2^2, & r > a \end{cases}$$

$$n_z(r) = n_t(r)(1+\delta)$$

and δ indicates the magnitude of anisotropy. The group velocity of various modes are calculated for anisotropic fibers with $n_1 = 1.5$ and $\Delta = 0.01$. Fig. 3 shows the difference of the group velocity for anisotropic fibers from that for the isotropic fiber ($\delta = 0$). The group velocity difference e_{an} is expressed as follows:

$$e_{an} = \frac{1}{C}|V_{gan} - V_g| \tag{20}$$

where V_{gan} and V_g indicate the group velocity for anisotropic and isotropic fibers, respectively. It becomes evident that the difference e_{an} increases about linearly with the magnitude of anisotropy δ. However, HE and EH modes are not shown in Fig. 3 because of their small group-velocity differences. The group velocity for TE modes is not affected by the variation δ of n_z. The group delay difference e_τ is expressed as

$$e_\tau / L \simeq \frac{n_1^2}{C} e_{an} \simeq 7.5 \times 10^3 e_{an} (\text{ns/km}). \tag{21}$$

For $\delta = 0.001$, the maximum group-delay deviation ($e_{an} \simeq 10^{-4}$) caused by uniaxial material is about 0.75 (ns/km). On the basis of this numerical results, we can know the effect of mechanical stress, e.g., high water pressure, on the group delay for square-law index fibers.

IV. Conclusions

A method for computing exactly the group velocity of propagation modes in optical fibers is described. In this vector analysis, optical fibers can contain uniaxial and dispersive material, and calculating the group velocity and the propagation constant requires relatively short running time on the computer, which is about twice as large as that in scalar analysis. The scalar approximation solutions of the group velocity are compared numerically with the results of the vector analysis to estimate the accuracy of the scalar approximation techniques. The deviation of the group velocity caused by uniaxial material is analyzed. It has become evident that this computation method is one of the most practical methods to calculate exactly the group velocity of guided modes.

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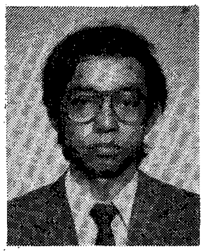

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Hollow Image Guide and Overlayed Image Guide Coupler

JING-FENG MIAO AND TATSUO ITOH, FELLOW, IEEE

Abstract —**A dielectric waveguide structure, hollow image guide, is described. This structure has several interesting characteristics useful for millimeter-wave applications. Dispersion characteristics and field distributions are theoretically and experimentally studied. The structure can also be considered as two parallel image guides coupled strongly by a dielectric overlay. Coupling characteristics between two image guide arms are studied numerically and experimentally.**

I. Introduction

RECENTLY, increasing attention has been paid to millimeter-wave circuits made of image-guide structures (Fig. 1(b)). It is often difficult to create a simple circuit for some signal processing functions with conventional image guides. Several attempts have been made to alleviate this difficulty [1], [2]. Both of these attempts modified the boundary conditions outside the dielectric rod of the waveguide. The new structure introduced in this paper modifies the interior of the dielectric rod. The structure may be called the hollow image guide (Fig. 1(a)). As shown in the figure, a portion of the dielectric material is removed from the rod (ϵ_2). The cross section of the hollow core (ϵ_1) has height h and width $2c$. The hollow core may be filled with another material if needed. The new waveguide can be used in conjunction with the techniques in [1] and [2]. However, the hollow image guide has a number of interesting features in its own right, making the design process more flexible. For instance, the hollow core may be used to control the field distribution outside the dielectric rod as well as the propagation constant without altering the exterior dimensions or the dielectric material. Also, by changing the core height gradually over some distance, we can create a smooth transition from the image guide to a solid-state device mounted in a hollow core.

The hollow image guide can also be thought of as two image guides (II in Fig. 1(a)) strongly coupled by way of a dielectric (I in Fig. 1(a)) or two image guides of height h coupled by an overlay of thickness of t. The degree of coupling can be adjusted by changing c, h, or b in Fig. 1(a).

In this paper, we study the propagation characteristics

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